# OPTIMIZATION OF A WAVE CANCELLATION MULTIHULL SHIP USING CFD TOOLS

C. Yang, R. Löhner and O. Soto

School of Computational Sciences, George Mason University
Fairfax VA 22030-4444, USA

## ABSTRACT

A simple CFD tool, coupled to a discrete surface representation and a gradient-based optimization procedure, is applied to the design of optimal hull forms and optimal arrangement of hulls for a wave cancellation multihull ship. The CFD tool, which is used to estimate the wave drag, is based on the zeroth-order slender ship approximation. The hull surface is represented by a triangulation, and almost every grid point on the surface can be used as a design variable. A smooth surface is obtained via a simplified pseudo-shell problem. The optimal design process consists of two steps. The optimal center and outer hull forms are determined independently in the first step, where each hull keeps the same displacement as the original design while the wave drag is minimized. The optimal outer-hull arrangement is determined in the second step for the optimal center and outer hull forms obtained in the first step. Results indicate that the new design can achieve a large wave drag reduction in comparison to the original design configuration.

## KEYWORDS

Hull form design, hull form optimization, wave cancellation multihull ship, trimaran, wave resistance, CFD tools, slender ship approximation, surface parameterization.

## INTRODUCTION

A small-water-plane area, tri-hull ship, termed the wave cancellation multihull ship (or trimaran), offers the possibility of dramatic wave drag reduction due to wave cancellation. Experimental evidence, e.g. by Wilson et. al. (1993) indicates that indeed these gains are achievable. From a design point of view, an important question that requires attention is how to deal with these types of multihull ships inside a general design framework. For example: should one optimize hull position and shape at the same time, or first obtain an optimal placement of hulls, followed by an optimal hull shape? The answer is not obvious.

As a first attempt to solve such a general design problem, a two-step design process is employed in the present paper. The optimal center and outer hull forms are determined independently in the first step, where each hull keeps the same displacement as the original design while the wave drag is minimized. The optimal outer-hull arrangement is determined in the second step for the optimal center and outer hull forms obtained in the first step. The present hull form optimization uses a gradient-based technique, which requires a field solution for each design variable. A very simple CFD tool based on the zeroth-order slender ship approximation is ideally suited for such an optimization technique because of its extreme simplicity and efficiency. It has been shown in Yang et. al. (2000) that this simple zeroth-order slender-ship theory, first given by Nobless (1983), is adequate for the purpose of determining the optimal hull arrangements for a wave cancellation multihull ship. This simple CFD tool has also been used with success for hull-form optimization in Letcher et. al. (1987) and Wyatt and Chang (1994).

The hull surface is represented by a triangulation. This triangulation can be used to evaluate the wave drag using present CFD tool. In order to obtain smooth hulls in the optimization process, the (very fast) pseudo-shell approach developed by Soto et. al. (2001) is employed. The surface of the hull is represented as a shell. The shell equations are solved using a stabilized finite element formulation with given boundary conditions to obtain the rotation and displacement fields. Almost every grid point on the hull surface can be chosen as design parameter, which leads to a very rich design space with minimum user input.

The optimal outer-hull arrangement for the optimal center and outer hulls is determined by searching the entire parameter space for each given Froude number for the purpose of minimizing the wave drag, that is evaluated very efficiently using the present simple CFD tool. Results indicate that the new design can achieve a fairly large wave drag reduction in comparison to the original design configuration.

## OPTIMAL SHAPE DESIGN

Any CFD-based optimal shape design procedure consists of the following ingredients:

- A set of design variables that determine the shape to be optimized;
- A set of constraints for these variables in order to obtain sensible shapes;
- An objective function $I$ to measure the quality of a design;
- A field solver to determine the parameters required by $I$ (e.g. drag, lift, moment, etc.);
- An optimization algorithm to determine how to change the design parameters in a rational and expedient way.

The present hull form optimization uses a gradient-based technique. The gradients are obtained via finite differences. This implies that for each design parameter, a field solution has to be obtained, making the use of extremely fast solvers imperative. One optimization step may be summarized as follows:

- Evaluate the objective function $I$ for the original geometry $\Sigma_s$ .
- Evaluate the gradient of the objective function for each design variable $k = 1, N_d$:
    a) Perturb the coordinates of the *k-th* design variable in its deformation direction by a small $\varepsilon$; the rest of design parameters are not moved;
    b) Solving the pseudo-shell problem using given boundary condition; this yields a new perturbed geometry $\Sigma_s^{'}$;
    c) Evaluate the objective function $I^{'}$ for the perturbed geometry $\Sigma_s^{'}$;

d) Obtain the gradient of the objective function with respect to the *k-th* design variable by finite differences as $(I - I') / \varepsilon$ .

- Make a line search in the negative gradient direction to find a minimum.

The detailed discussion about this approach can be found in Soto et. al. (2001). In the sequel, we describe the objective function, the surface representation and the CFD solver used.

## OBJECTIVE FUNCTION

From an engineering perspective, it is important to reduce the wave drag while still being able to achieve a given displacement. For this reason, the objective function used for hull shape optimization is given by:

$$I = \omega_1 \frac{C_W}{C_W^*} + \omega_2 \frac{|(V^* - V|}{V^*}$$

where $C_W$ and $C_W^*$ are the wave drag coefficient and its initial value, $V$ and $V^*$ the hull displacement and its initial value, and $0<\omega_1, \omega_2<1$ are relative weights. It was found to be very important to cast the optimization function in this non-dimensional form. Otherwise the weights $\omega_{1,2}$ have to be adjusted for different geometries.

## SURFACE REPRESENTATION

There are many ways to represent surfaces. Analytical expressions given by B-Splines, NURBS or Coon's patches are common. Another possibility is to take a surface triangulation and then allow every point on the surface to move. This discrete surface representation can always be obtained from analytical surface descriptions, and, for sufficiently fine surface triangulations, provides a very rich design space with minimal user input. For this reason, this discrete surface description is used in the present work.

During optimization, the individual points on the surface may move in such a way that a non-smooth hull is produced. In order to obtain smooth hulls, the (very fast) pseudo-shell approach developed by Soto et. al. (2001) is employed. The surface of the hull is represented as a shell. The movement of points is recast as a forcing term for the movement of the shell. The shell equations are solved using a stabilized finite element formulation with given boundary conditions to obtain the rotation and displacement fields. The boundary conditions in a shape optimization problem are dictated by the design parameter displacement and the geometrical constrains. In the optimal design process, the user only needs to generate the original surface mesh and a few design variables. The rest of the design parameters and their respective deformation modes can be generated automatically by the method.

## CFD SOLVER FOR WAVE DRAG REPRESENTATION

Consider a ship advancing along a straight path, with constant speed $U$, in calm water of effectively infinite depth and lateral extent. The $x$ axis is taken along the path of the ship and points toward the ship bow, the $z$ axis is vertical and points upward, and the mean free surface is the plane $z=0$. Non-dimensional coordinates $(x, y, z)$ and velocities $(u, v, w)$ are defined in terms of a characteristic length $L$ (taken as the length of the center hull for a wave cancellation multihull ship) and the ship speed $U$. The wave drag $C_W$ is evaluated using the Havelock formula

$$C_W = \frac{D_W}{\rho\ U^2 L^2} = \frac{\upsilon}{2\pi} \int_{-\infty}^{\infty} \frac{k\, d\beta}{k-\upsilon} \quad (S_r^2 + S_i^2) \tag{1a}$$

for the energy radiated by the far-field waves. $D_W$ is the wave drag and $\nu$ is defined as

$$\upsilon = \frac{1}{2F^2} \qquad with \qquad F = \frac{U}{\sqrt{gL}} \tag{1b}$$

Furthermore the wavenumber $k$ in Eqn. 1a is defined in terms of the Fourier variable $\beta$ as

$$k(\beta) = \upsilon + \sqrt{\upsilon^2 + \beta^2} \tag{1c}$$

$S_r$ and $S_i$ are the real and imaginary parts of the far-field spectrum function $S = S(\alpha, \beta)$ where $\alpha$ is defined in terms of the Fourier variable $\beta$ as

$$\alpha(\beta) = \sqrt{k(\beta)} \,/\, F \tag{1d}$$

This relation and expression Eqn. 1c follow from the dispersion relation $F^2\alpha^2 = k$.

The wave spectrum function $S = S_r + i\, S_i$ in the Havelock integral (Eqn. 1a) is approximated here by the zeroth-order slender ship approximation defined in Noblesse (1983) as

$$S = \int_\Sigma n^x e^{k\,z + i(\alpha\, x + \beta\, y)} dA + F^2 \int_\Gamma (n^x)^2 t^y e^{i(\alpha\, x + \beta\, y)} dL \tag{2}$$

Here, $dA$ and $dL$ stands for the differential elements of area and arc length of the mean wetted hull surface $\Sigma$ and the mean waterline $\Gamma$, and $n^x$ and $t^y$ are the $x$ and $y$ components of the unit vectors, $\vec{n} = (n^x, n^y, n^z)$ and $\vec{t} = (t^x, t^y, 0)$, normal to the ship hull surface $\Sigma$ and tangent to the ship waterline $\Gamma$; $\vec{n}$ points inside the flow domain (i.e. outside the ship) and $\vec{t}$ is oriented clockwise (looking down). Thus the wave spectrum function $S$ in the Havelock formula for the wave drag is defined explicitly in terms of the ship speed and the hull form in the zeroth-order slender ship approximation.

The present wave cancellation multihull ship (see Wilson et. al. (1993) and Yang et. al. (2000)) consists of one main center hull centered at $(0, 0, 0)$ and two identical outer hulls centered at $(a, \pm b, 0)$. In the first step of the optimal design process, the wave drag for each individual hull is evaluated using Eqns. 1-2, and the center hull and the outer hull are optimized independently for the purpose of minimizing the wave drag of each hull.

In the second step of the optimal design process, the total wave drag $C_W$ for the optimal center and outer hull forms obtained form the first step needs to be computed so that an optimal arrangement of the outer hull with respect to the center hull can be determined. The total wave drag $C_W$ for such a wave cancellation multihull ship can be expressed as (see Yang et. al. (2000))

$$C_W = C_W^c + 2C_W^o + C_W^i \tag{3a}$$

where $C_W^c$ and $C_W^o$ are given by

$$C_W^c = \frac{\upsilon}{2\pi} \int_{-\infty}^{\infty} \frac{k\, d\beta}{k-\upsilon} \quad (S_r^{c^2} + S_i^{c^2}) \tag{3b}$$

$$C_W^o = \frac{\upsilon}{2\pi} \int_{-\infty}^{\infty} \frac{k\, d\beta}{k-\upsilon} \quad (S_r^{o^2} + S_i^{o^2}) \tag{3c}$$

and represent the wave drags of the center hull and of an outer hull, respectively. The spectrum functions $S^c = S_r^c + iS_i^c$ and $S^o = S_r^o + iS_i^o$ are defined by Eqn. 2 in which $\Sigma$, $\Gamma$ are taken as $\Sigma_c$, $\Gamma_c$ or $\Sigma_o$,

$\Gamma_o$, i.e., the wetted surface and waterline of the center hull or the outer hull. The component $C_W^i$ accounts for interference effects and is defined as

$$C_W^i = \frac{\upsilon}{2\pi} \int_{-\infty}^{\infty} \frac{k\, d\beta}{k-\upsilon} \quad \{\frac{A}{2}\cos(2b\beta\ ) + A^R \cos(a\alpha\ )\cos(b\beta\ ) + A^I \sin(a\alpha\ )\cos(b\beta\ )\} \tag{3d}$$

where $A$, $A^R$ and $A^I$ are defined as

$$\begin{Bmatrix} A \\ A^R \\ A^I \end{Bmatrix} = \begin{Bmatrix} (S_r^o)^2 + (S_i^o)^2 \\ S_r^c S_r^o + S_i^c S_i^o \\ S_i^c S_r^o - S_r^c S_i^o \end{Bmatrix} \tag{3e}$$

It is noted that the wave spectrum functions $S^c$ and $S^o$ are independent of the parameters $a$ and $b$ within the current approximation. Therefore, the wave spectrum functions $S^c$ and $S^o$ defined by Eqn. 2 and the related functions $A$, $A^R$ and $A^I$ given by Eqn. (3e) similarly need to be evaluated only once per Froude number.

## NUMERICAL RESULTS AND CONCLUSIONS

The first design problem considered here is to determine the optimal hull arrangement for the original wave cancellation multihull ship for the purpose of minimizing the wave drag of the ship. Figure 1 depicts the experimental values of the residuary drag coefficient $C_R$ given in Wilson et. al. (1993) and the corresponding predictions of the wave-drag coefficient $C_W$ given by the present method for the four hull arrangements. These hull arrangements correspond to $a$=-0.128, -0.205, -0.256, -0.385 and $b$=0.136 for all four cases. In this figure, the $C_R$ and the $C_W$ are nondimensionalized in terms of the surface area of the wetted hull, in the usual fashion. In the rest of the figures, the $C_W$ is defined by Eq. 1a. It can be seen that the $C_W$ predicted by the present method is in fair agreement with the experimental $C_R$. In particular, the variation of $C_R$ with respect to the Froude number $F$ is well captured by the theory. The present method may therefore be used for the purpose of determining the optimal arrangements of the outer hulls.

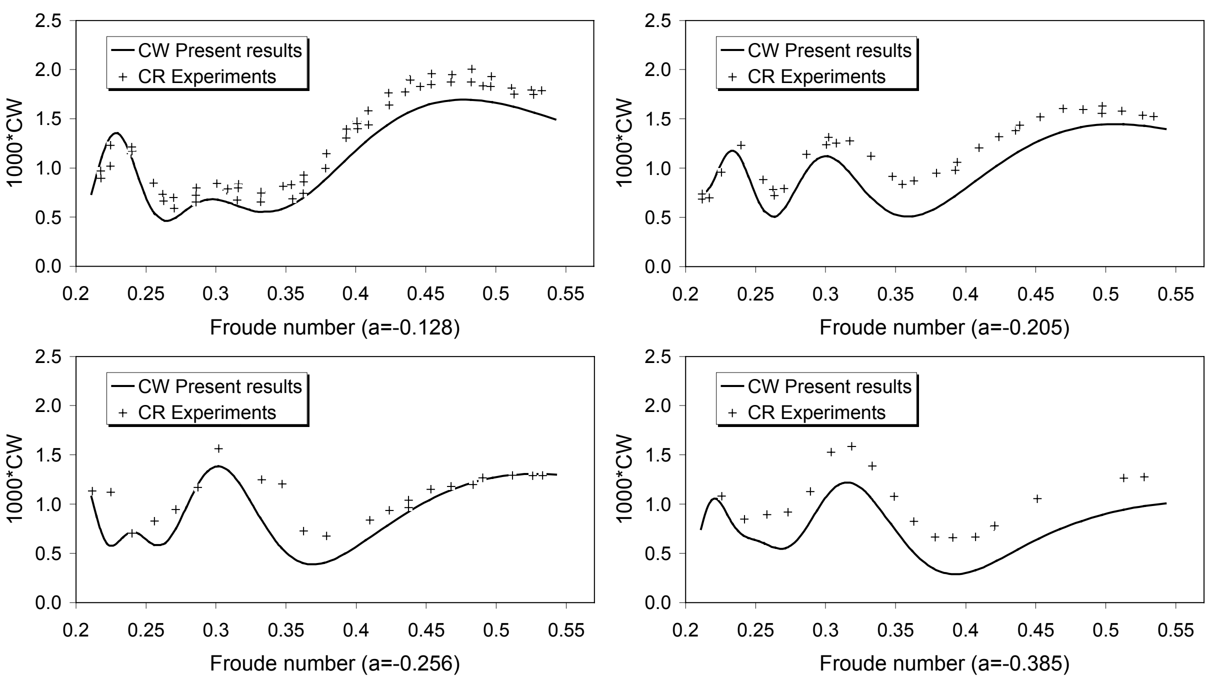
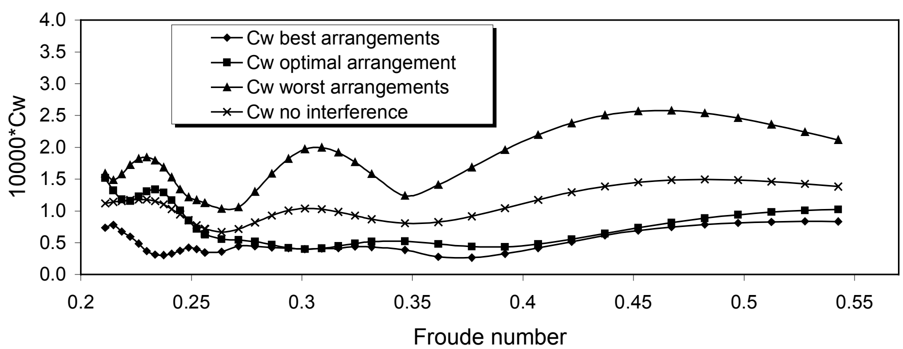



Fig. 1. Calculated wave drag and experimental residuary drag

The hull arrangements within the range of $-0.75 \le a \le 0.75$ and $0.1 \le b \le 0.3$ with $\Delta a = 0.025$ and $\Delta b = 0.01$ are studied for 38 values of Froude numbers with $0.2147 = F_1 \le F_i \le F_{38} = 0.5426$. For $a$=0.75, the sterns of the outer hulls are aligned with the bow of the main center hull; similarly, the bows of the outer hulls are aligned with the stern of the center hull if $a$=-0.75. This study represents $61 \times 21 = 1281$ hull arrangements and $61 \times 21 \times 38 = 48678$ evaluations of $C_W$. The optimal hull arrangement for the speed range $F_1 \le F_i \le F_{38}$ approximately corresponds to $a$=0.55, $b$=0.11. Fig. 2 depicts the variations, with respect to the Froude number $F$, of the "no-interference wave-drag coefficient", $C_W^c + 2C_W^o$ and of the wave-drag coefficients $C_W^{best}$ and $C_W^{worst}$ associated with the best and worst hull arrangements found within the region. Fig. 2 also shows the wave drag-coefficient curve $C_W^{optml}(F)$ for the optimal hull arrangement ($a$=0.55, $b$=0.11). The wave-drag coefficient curve $C_W^{optml}(F)$ corresponds to a hull arrangement that remains fixed over the entire speed range, while the curves $C_W^{best}(F)$ and $C_W^{worst}(F)$ are associated with hull arrangements that vary with speed. The large differences between $C_W^{best}$ and $C_W^{worst}$ apparent in Fig. 2 demonstrate the importance of selecting




favorable hull arrangements. Indeed, Fig. 2 shows that the ratio, $C_W^{best} / C_W^{worst}$, approximately varies between 2 and 6 within the speed range considered. Fig. 2 also shows that the wave-drag curve, $C_W^{optml}(F)$, is significantly lower than the curve $C_W^{worst}(F)$ corresponding to the worst hull arrangements, and even the curve corresponding to the no-interference wave drag $C_W^c + 2C_W^o$, over most of the speed range. In fact, the curve $C_W^{optml}(F)$ is remarkably close to the curve $C_W^{best}(F)$ corresponding to the best hull arrangement at every speed, over a broad speed range.

Fig. 2. Wave drag coefficient for different hull arrangements

The second design problem considered here consists of two steps. The optimal center and outer hull forms are determined independently in the first step using present optimization technique. The center and outer hulls of the original wave cancellation multihull ship are used, respectively, as starting baseline hulls in the optimization cycles. During the optimization process, each hull keeps the same displacement as the original design while the wave drag is minimized. There are 76 design variables for the center hull and 64 for the outer hull. Four design cycles are required for each hull form optimization.

The optimal center hull ($A$) is obtained by minimizing $C_W^c(F)$ for one Froude number, $F$=0.5, and the optimal center hull ($B$) is obtained by minimizing $C_W^c(F)$ for five values of Froude number, $F$=0.3, 0,35, 0.4, 0.45, 0.5. Fig. 3a depicts the predicted wave-drag-coefficient curves corresponding to the original center hull and two optimal center hulls. Fig. 3a indicates that the wave drag associated with the optimal center hull ($A$) is reduced tremendously in comparison to the original center hull when the Froude number is above 0.4. However, the wave drag is increased slightly at lower Froude numbers. Fig. 3a also indicates that the wave drag associated with the optimal center hull ($B$) is reduced over almost the entire speed range in comparison to the original center hull. As expected, the wave drag reduction for the optimal center hull ($B$) at high Froude numbers is not as pronounced as that for the optimal center hull ($A$). Similarly, the optimal outer hull ($A$) is obtained by minimizing $C_W^o(F)$ for one Froude number, $F$=0.35, and the optimal center hull ($B$) is obtained by minimizing $C_W^o(F)$ for two values of Froude number, $F$=0.3, 0,35. Fig. 3b depicts the predicted wave-drag-coefficient curves corresponding to the original outer hull and two optimal outer hulls. Fig. 3b indicates that the optimal outer hull ($B$) has a larger wave drag reduction than that of the optimal outer hull ($A$) over almost the entire speed range in comparison to the original outer hull. Therefore, the optimal outer hull ($B$) and the optimal center hulls ($A$) and ($B$) will be used further on as two optimal hull design cases for determining the optimal hull arrangements. The optimal center and outer hulls are shown in Fig. 4.

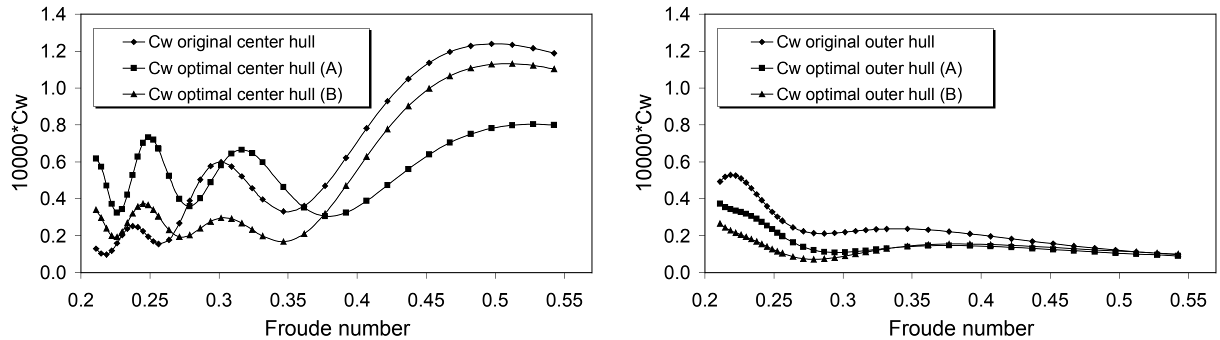



Fig. 3. Wave drag coefficient for original and optimal hulls

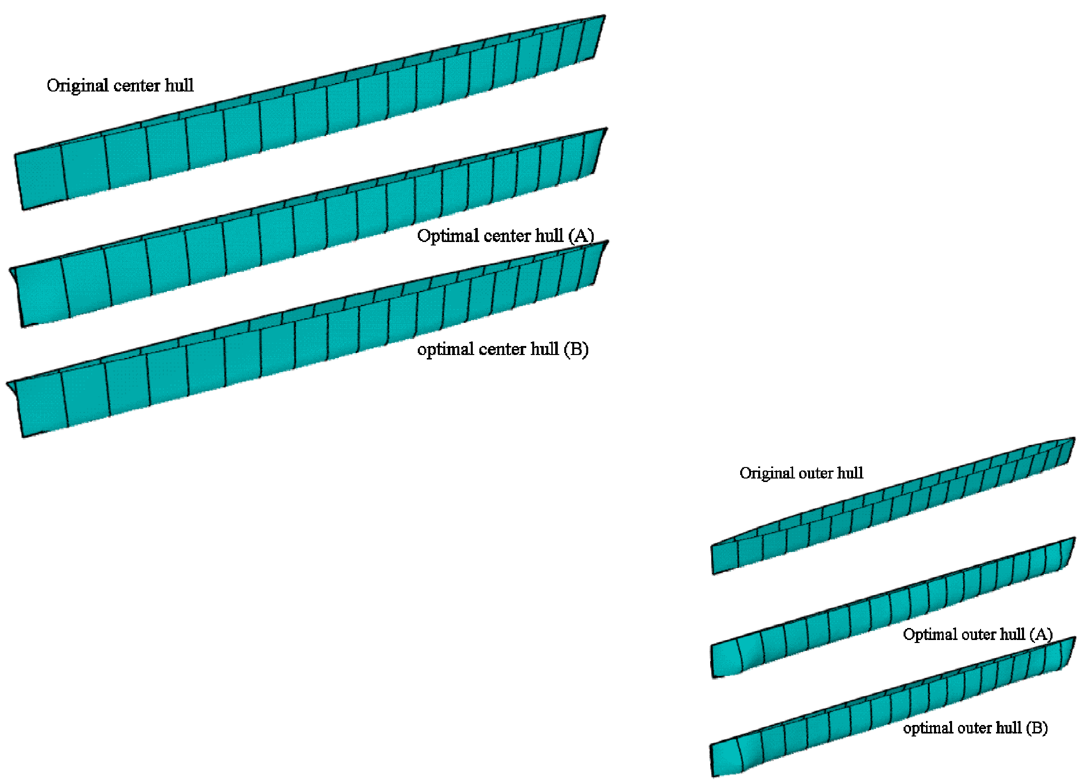



Fig. 4. Original and optimal center and outer hulls

The optimal hull arrangement is determined in the second step of the design process using the hull forms obtained in the first step of this design problem. The same methodology and notations, described in the first example, are used hereafter for the combinations of two optimal center hulls and one optimal outer hull, i.e., optimal hull (*A*) (optimal center hull (*A*) and optimal outer hull (*B*) ) and optimal hull (*B*) (optimal center hull (*B*) and optimal outer hull (*B*) ), for the purpose of minimizing the wave drag of each new wave cancellation multihull ship. The optimal hull arrangements for the optimal hull (*A*) and (*B*) approximately correspond to $a$=0.65, $b$=0.11 and $a$=0.60, $b$=0.11, respectively. Fig. 5 depicts the variations, with respect to the Froude number $F$, of the computed wave-drag coefficients associated with the optimal hull arrangements obtained for the original hull ($a$=0.55, $b$=0.11) and optimal hull (*A*) ($a$=0.65, $b$=0.11) and optimal hull (*B*) ($a$=0.60, $b$=0.11), and the wave drag coefficients associated with the experimental arrangements for the original hull. Fig. 5 indicates that the optimal hull (*A*) can reach large wave drag reduction when the Froude number is above 0.4, and the optimal hull (*B*) can achieve noticeable drag reduction for almost the entire speed range. Fig. 5 also shows that the fourth experimental arrangement ($a$=-0.385, $b$=-0.136) are the best one for the purpose of minimizing the wave drag at higher Froude numbers in comparison to the other three experimental arrangements. Fig. 7 depicts the wave drag reduction for the optimal designs of the original hull, optimal hull (*A*) and optimal hull (*B*) with respect to the fourth experimental arrangement ($a$=-0.385, $b$=-0.136) of the original hull. This figure shows that these three designs can reduce drag for almost the entire speed range. The maximum wave drag reductions for these three designs are approximately 20%, 56% and 40% in high-speed range, and 71%, 76% and 87% in low- speed range.

In summary, the present simple CFD tool, coupled to a discrete surface representation and a gradient-based optimization procedure, can be used very effectively for the design of optimal hull forms and optimal arrangement of hulls for a wave cancellation multihull ship. Results indicate that the new design can achieve a fairly large wave drag reduction in comparison to the original design configuration. This optimization technique can be readily used in the routine single hull or multihull ship design to minimize the wave drag.

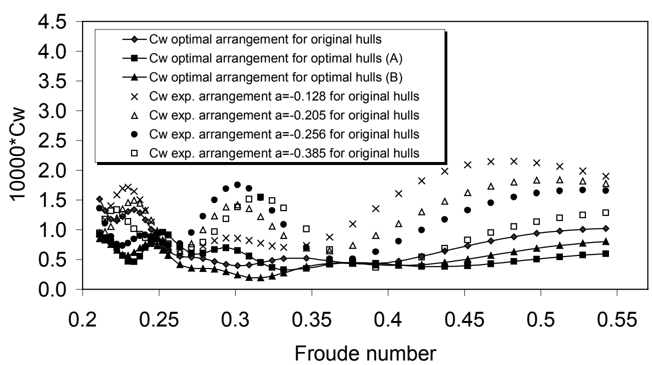



Fig. 5. Wave drag coefficient for different hull forms and hull arrangements

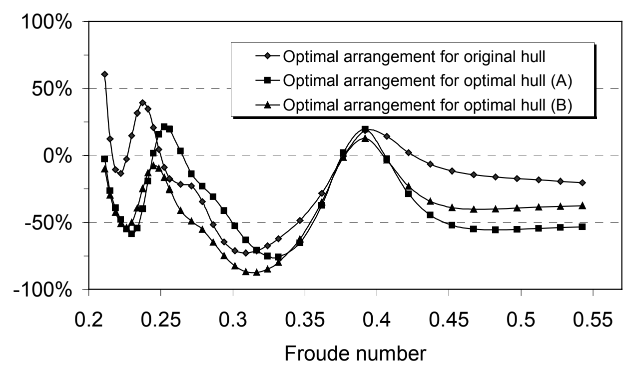



Fig.6. Wave drag reduction for different optimal hull arrangements

**ACKNOWLEDGEMENTS**

This work was partially funded by AFOSR (Dr. Leonidas Sakell technical monitor) and by NRL (Dr. William Sandberg technical monitor). All computer runs were performed on a 128-Processor R10000 SGI Origin 2000 at NRL.

***References***


Letcher, J.S. Jr., Marshall, J.K., Oliver, J.C. III and Salvesen, N. (1987) Stars & Stripes. *Scientific American*, **257**: 34-40.

Noblesse, F. (1983). A slender-ship theory of wave resistance. *Journal of Ship Research*, **27**: 13-33.

Wilson, M.B., Hsu, C.C. and Jenkins, D.S. (1993). Experiments and predictions of the resistance characteristics of a wave cancellation multihull ship concept, *23rd American Towing Tank Conf.*, 103-112.

Soto, O., Löhner, R. and Yang, C. (2001) A Stabilized pseudo-shell approach for surface parameterization in CFD design problems, submitted to *Comm. In Num. Method in Engineering*.

Wyatt, D.C. and Chang, P.A (1994). Development and assessment of a total resistance optimized bow for the AE 36. Marine Technology, **31**: 149-160.

Yang, C., Noblesse, F., Löhner, R., Hendrix, D. (2000). Practical CFD applications to design of a wave cancellation multihull ship. *23rd Symp. On Naval Hydrodynamics*, Val de Reuil, France.